# ON THE JUDGMENT OF ANGLES AND POSITIONS OF LINES.

## *A*.—On the Judgment of Angles.

By Joseph Jastrow, Ph. D.

With the assistance of Geo. W. Moorehouse, Fellow in Psychology.

The nature and extent of our errors in estimating and reproducing angles are the subject of our present inquiry. The point acquires a special interest from the fact that a number of writers have based their explanations of important optical illusions upon the view (apparently not tested by experiment) that acute angles are underestimated and obtuse angles overestimated. Such an investigation naturally begins with a definite mode of judgment or reproduction under definite circumstances, and is then to be supplemented by an investigation of the extent to which the results obtained are due to peculiarities of the method employed. We selected the angles 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165° as our standard angles and drew these upon circular pieces of cardboard (3 in. in diameter); the lines themselves were 30 mm. long and were so placed that one line was always horizontal. The papers on which the angles were to be reproduced by the subject were placed on a table and the drawings made in the normal writing position. For convenience all acute angles were formed on the left hand side and obtuse ones on the right, so that the drawing might be uniformly from left to right. The subject viewed the angle as long as was needed to fix it in his mind (from 5 to 15 seconds) and immediately thereupon, from his memory of the angle, drew another as nearly as possible equal to the first. He did this by adding a line to a horizontal line of 30 mm. which was given him (always in the same position) upon the squares of paper upon which he drew. The drawing was done with a hard, well-pointed lead-pencil. Each of the standard angles occurred twice in a set of 22 angles, the order of the angles being determined by chance. The results of the measurements of 62 such sets, or 124 reproductions of each angle by 13 subjects in all, are shown in the following table :

| Standard Angle | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Average Reproduction | 18°03′ | 30°27′ | 42°09′ | 56°25′ | 69°44′ | 90°06′ | 113°34′ | 123°55′ | 136°29′ | 150°33′ | 163°49′ |
| Error (from corrected standard)[1] | +2°23′ | -0°56′ | -2°16′ | -5°13′ | -6°46′ | +0°06′ | +7°54′ | +3°22′ | +0°19′ | -1°05′ | -2°41′ |

[1] For details of measurement and preparation of tables and curves, consult the note at the end.

It will be seen that the overestimations and underestimations can hardly be said to follow any simply formulatable law, such as the underestimation of acute and the exaggeration of obtuse angles; their full significance appears only in the curve as given below (Fig. 1); in this curve the differences between the actual and the re-

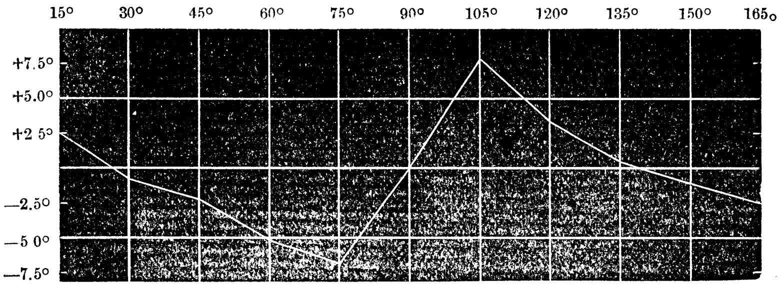


Fig. 1. Errors, in degrees, in reproducing angles by memory: Average of 13 subjects.

produced angles are plotted to the scale of one of the divisions to five degrees of error. This is our general result and its salient characteristics are, (*a*) the exaggeration of the angle at 15°, which passes into a gradually increasing underestimation up to 75°; (*b*) the correct reproduction of right angles; (*c*) the maximum exaggeration of the angle of 105°, which is followed by a decreased exaggeration, passing into an underestimation of very obtuse angles.

The next important inquiry is naturally how far the result is typical, and how far accidental; how far the result of the combination of different curves and how far the individual records agree with the general result.

By each of the thirteen individuals the angle of 15° is exaggerated; in all of the thirteen cases there is a falling off towards the next point 30°, the angle being about as frequently slightly overestimated as slightly underestimated; in all but two cases there is a fairly regular increase of the underestimation, reaching a maximum at 75°; in all cases the right angle is nearly correctly reproduced, the error being as often in one direction as in the opposite; in all cases the curve then sharply rises, reaching the maximum of exaggeration at 105°, and from there in eleven cases there is a more or less regular decline; the curve at the last point 165° falling below the line. Again, we may calculate the average deviation of the thirteen results from their mean; this for the eleven angles is 2° 31′, and of the 143 records (13 subjects for 11 angles) 87 show a deviation less than this average. Regarding the variation for the different angles it is least for 90°, the other angles following in this order, 150°, 165°, 15°, 30°, 135°, 75°, 45°, 120°, 105°, 60°.

Comparing the general outlines of the individual curves with the average curve, we find that in ten of the thirteen individuals, the correspondence is obvious and in most of these, striking; in one case the curve presents quite a different appearance, and in two other cases the differences are considerable. This diverging curve, however, is that of a professor of engineering, who has considerable experience in the estimation of angles; he draws angles of 30°, 90°, 135° and 165° very accurately, overestimates angles of 15°, 60°, 105° and 150° considerably, and underestimates the angles of 45°, 75° and

120°, thus presenting a zig-zag curve. The other two subjects came prepared with some practice in drawing and exhibited peculiarities in the mode of estimating the angles. In general there is thus a very striking similarity between the individual and the general result, so that the curve may be regarded as fairly typical for the average person.

The suggestion is close at hand that this result may be influenced by the mode of reproduction; to test this, three of the subjects reproduced the angles, not from memory but with the standard angle constantly visible for comparison. The resulting curve is quite similar in the two cases, the essential difference being that the entire curve is closer to the true line of no error, i. e., the error is smaller. Fig. 2 shows the average result of the three subjects for each mode of reproduction.

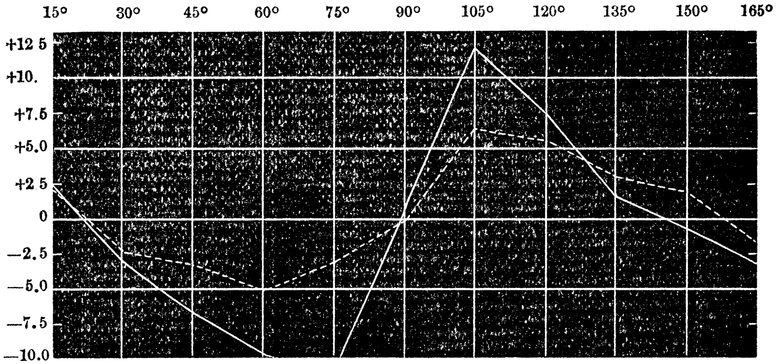


FIG. 2.

......Errors in reproducing angles with both angles visible. Average of three subjects.
———Errors in reproducing angles by memory. Average of same three subjects.

It is likewise interesting to determine the regularity and accuracy of these methods of reproducing angles; a sufficient test of this is the average deviation of the results from their mean. This is very troublesome to calculate and we have contented ourselves with doing it for two individuals. In reproducing angles by memory the average deviation of the one subject (fourteen judgments of eleven angles) was 2° 50′, of the other 3° 05′; in reproducing angles with the standard angle in sight 1° 33′ for the one and 2° 08′ for the other. There is thus indicated an increase of regularity with the decrease of absolute error. A comparison of deviations for the several angles shows that the right angle has by far the smallest deviation (about one-fourth of the average), and that the smallest and largest angles are somewhat more regularly reproduced than intermediate ones, and thus again indicate the direct relation between error and regularity.

Finally as to the significance of these results, we may offer the following suggestion. Our curve may be viewed as consisting of two portions, the first beginning with 15° and ending with 75°; the second beginning with 105° and ending with 165°, i. e., we omit the right angle as well as the angle 0°. In that case the curve falls into two (often strikingly) similar portions, beginning with an exaggeration and ending with an underestimation. This would mean that angles with a small excess over 0° or 90° are more exaggerated

or less underestimated than angles with a greater excess over 0° or 90°, and in this special sense is it true that acute angles are underestimated and obtuse angles overestimated; the smallest and largest angles forming an exception to the generalization.

*More on Details of Method.*—The angles were measured by applying a square of card-board 25 mm. square to the horizontal line, having the apex of the angle coincide with a corner of the square; the distance of the intersection of the oblique line with the side of the square was noted to the nearest ¼ mm. and from this the tangent of the angle could be readily calculated; for the above process gave us the measurement of the opposite side of an angle whose adjacent side was always 25 mm. In this way under favorable circumstances a set of twenty-two angles could be measured and the results tabulated in five minutes. There is inevitably some error in this mode of measuring, and to eliminate such error, as far as possible, we measured our standard angle by the same method, finding as a result the angles 15° 40′, 31° 23′, 44° 25′, 61° 38′, 76° 30′, 90°, 105° 40′, 120° 33′, 136° 10′, 151° 38′, 166° 30′; the deviations plotted in the curves are from these angles and not from the theoretical angles 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°. In comparing the general average, the average of each individual was weighted by the number of sets of which it was the average.

## *B.*—On the Judgment of the Positions of Lines.

With the assistance of James H. Turner.

Several points in the results just described suggested further research. The fact that one side of each angle was given as well as that the lines are drawn on square pieces of paper with one line parallel to the side of the square, may have important influences upon the results. To eliminate entirely the influences which these conditions may have induced, it seemed necessary to ensure an environment for the subject in which no straight lines whatever should be visible except those judged, for the lines of the floor and walls are manifestly sufficient to give him his vertical and horizontal and thus a basis for estimating angles.

To secure these conditions we arranged the experiment so that the subject could see nothing but one or two white card-board discs, four inches in diameter, upon which was drawn a straight black line three inches long. The two discs, one above the other, were viewed against a large black disc thirty inches in diameter, all placed in a vertical position. Above his head was a parasol-like frame, from which hung black draperies and a similar black cloth was drawn across his lap. When in position he was completely enclosed under this canopy, the light coming in from the back above his head; no portion of the floor or walls was visible to him. He was seated on a chair with his eyes on a level with a point midway between the two discs and about 15-20 inches away from them. The subject's arms reached outside of this canopy and held two handles attached by cords to the axle upon which the upper of the small discs turned. By pulling the right or left cord the subject could thus bring the straight line drawn upon the card-board into any position. This line was three inches long and 1 mm. wide and its centre was the centre of the four-inch disc. At the other end of this apparatus, which was firmly mounted upon a table, sat the observer, who had before him two circles divided to half degrees; to each of the other ends of the axles, upon one end of which was glued a four-inch disc, was attached a "hand" ending in a fine needle point, which was so ad-

justed as to assume precisely the same position as the line upon the disc. This adjustment was constantly regulated by setting the line vertical (by a fine plumb line) and noting the deviation, if any, of the hand from 90°. Finally a third axle midway between the two that bore the four-inch discs and thus in the centre of the thirty inch disc, bore a five-inch black paper disc eccentrically mounted and covering at pleasure either the upper or the lower disc.

A twisted cord attached to the axle and also to a hinged lever, the cord drawn and kept tense by a weight, enabled the operator by a simple movement to conceal either the upper or lower disc. Both were never in view at once. With this apparatus our method of experimentation is very simple. The operator sets the line of the lower disc at any desired angle; he then uncovers this disc, allowing the subject to view it until a clear impression of the position of the line is obtained; he then instantly covers this disc, and the subject, by means of the strings, sets the line of the upper disc to correspond to the remembered position of the lower line. The operator reads the position on his divided circle and the difference in the two readings gives the error. In the meantime another position has been set on the (invisible) lower disc, which is now revealed, and so on. Eighteen positions were used, forming angles of 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°, 100°, 110°, 120°, 130°, 140°, 150°, 160°, 170° respectively with the horizon. The observations were taken in sets of eighteen, each angle occurring once in a set. With this apparatus we studied the error in setting, after a brief interval, one line in the same position as a standard line; this judgment clearly involves angles, for it is based partly at least upon the angles formed with ideal verticals and horizontals. It also involves the conception of parallelism, for the task may be conceived as that of setting one line parallel to another.

Ten individuals were tested, seven of them drawing each position of the line ten times, and three of them each line twenty times. The average settings in degrees and tenths of degrees of each of the ten subjects for each of the eighteen positions of lines are exhibited in the following table. The Roman numbers indicate the different subjects; the upper line, the standard positions of the lines. The average of all is shown in the lowest line:

| | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° | 100° | 110° | 120° | 130° | 140° | 150° | 160° | 170° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I. | 9.6 | 23.5 | 31.7 | 37.8 | 49.0 | 60.8 | 69.7 | 78.7 | 91.2 | 104.2 | 113.7 | 122.5 | 131.7 | 140.9 | 153.7 | 162.0 | 171.5 | 178.5 |
| II. | 12.0 | 21.1 | 27.8 | 41.4 | 55.5 | 60.2 | 68.3 | 79.2 | 92.0 | 104.2 | 114.4 | 123.4 | 129.8 | 140.8 | 151.6 | 162.5 | 171.0 | 180.4 |
| III. | 10.2 | 19.9 | 29.1 | 41.9 | 52.0 | 61.4 | 74.1 | 83.8 | 91.2 | 103.6 | 111.9 | 124.8 | 131.9 | 146.6 | 155.9 | 167.2 | 176.2 | 181.4 |
| IV. | 9.6 | 18.4 | 28.5 | 40.5 | 47.9 | 55.9 | 63.9 | 71.8 | 90.6 | 105.2 | 111.9 | 121.1 | 131.2 | 144.9 | 153.7 | 162.5 | 174.4 | 180.7 |
| V. | 11.9 | 19.8 | 30.0 | 43.3 | 50.9 | 57.9 | 67.6 | 74.9 | 90.4 | 102.2 | 111.0 | 121.2 | 129.7 | 138.3 | 150.8 | 157.2 | 166.4 | 179.6 |
| VI. | 14.3 | 24.6 | 37.8 | 47.1 | 54.4 | 65.3 | 72.0 | 82.8 | 90.9 | 106.2 | 114.4 | 123.6 | 133.2 | 141.2 | 153.1 | 161.3 | 172.7 | 180.6 |
| VII. | 10.8 | 21.9 | 31.9 | 39.4 | 52.4 | 61.0 | 72.7 | 82.9 | 90.3 | 101.5 | 114.0 | 121.4 | 131.7 | 143.2 | 152.7 | 162.5 | 173.0 | 180.3 |
| VIII. | 9.5 | 24.3 | 31.6 | 37.8 | 49.8 | 60.6 | 70.0 | 82.7 | 90.8 | 99.2 | 110.2 | 117.1 | 134.2 | 141.4 | 148.7 | 162.9 | 174.5 | 181.7 |
| IX. | 9.2 | 21.1 | 33.5 | 44.2 | 53.6 | 62.5 | 73.0 | 81.0 | 89.6 | 100.7 | 114.8 | 123.4 | 132.5 | 141.8 | 155.6 | 161.2 | 170.3 | 180.2 |
| X. | 10.6 | 19.6 | 29.8 | 39.0 | 46.0 | 57.8 | 66.7 | 76.1 | 89.0 | 105.0 | 114.7 | 126.1 | 133.5 | 141.9 | 151.3 | 158.5 | 168.9 | 179.8 |
| Average | 10.77 | 21.42 | 31.7 | 41.24 | 51.4 | 60.34 | 69.80 | 79.39 | 90.6 | 103.2 | 113.1 | 122.46 | 131.94 | 142.1 | 152.91 | 161.78 | 171.89 | 180.32 |

If these averages be plotted in a curve similar to that drawn for reproduction of angles, a very irregular curve will result, presenting practically no constant characteristics. The errors vary with each angle and almost with each individual. All the angles are so nearly correctly reproduced that the order of their correctness seems almost accidental, although there are abundant indications that the vertical (90°) and the horizontal (180°) are more accurately reproduced than any others.

It is also true that the angles are rather more apt to be over-estimated than under-estimated, and that the obtuse angles are rather more over-estimated than the acute ones. Of the 180 records entered in the table, 138 are over-estimations and 42 under-estimations. Of the 90 records for angles of 90° and less, 55 are over-estimations and 35 under-estimations; of the 90 records for angles between 90° and 180°, 83 are overestimations and 7 underestimations. To this extent the characteristics of the former results reappear. As a very rough comparison of the errors in drawing angles from memory with one angle and the side of the other given, in doing this with both angles visible, and in judging positions of lines, it may be stated that the average error for all angles (without regard to their being positive or negative) of the three individuals whose records we have for all those methods are 3° 40′ in the first case, 2° 42′ in the second, and 1° 53′ in the third. The entire curve for positions of lines is thus nearer the line of no error than that for reproducing angles. It must be remembered that much of this resulting absence of error is due to the balancing of errors of opposite directions, particularly so with acute angles. The subject had, if anything, a smaller degree of confidence in the correctness of his reproduction of positions of lines than in those of angles, and had decidedly less confidence in the former than in the latter when these were drawn with both angles visible. An indication of the regularity of these reproductions is furnished by the average deviations of the individual reproductions from their mean. This for the three methods (average of all the eleven angles, eighteen positions) is for one individual 2° 50′ for angles from memory, 1° 33′ for angles with the standard angle visible and 3° 15′ for positions of lines; for another subject 3° 05′, 2° 08′ and 3° 49′. This would indicate the greatest variability, least regularity for the estimation of positions of lines, the least variability for reproduction of angles, with the standard angle visible, and an intermediate degree for reproductions of angles from memory. This order corresponds with the subjective feeling.

In general, then, we conclude that in the reproduction of positions of lines without reference to any but imagined coördinates, the absolute error is small; is on the whole an overestimation of the angle;[1] is greater with obtuse than with acute angles; while the individual variation of the results is rather large. No simply formulatable law is followed by the resulting curve of error; the errors varying irregularly with the angle and the individual.

### *B II.*—On the Judgment of Horizontal, Vertical and Oblique Positions of Lines.

With the assistance of W. D. Brown.

It has already appeared that much of our perception of angles and positions of lines takes place by reference to an ideal vertical and horizontal which we constantly carry with us and have had

[1] This overestimation means that the upper end of the line was set too far to the right; this may be due to a greater dependence of the right eye in judging or in the adjustment of the right hand.

forcibly impressed upon us by the countless verticals and horizontals with which civilization has surrounded us. It would indeed be strange if this enormously extensive experience with right angles, verticals and horizontals should not have left its impress upon our psycho-physiological organism. We have had some evidence of it in the accuracy of judging right angles; and the importance of the subject led us to undertake the determination of the accuracy of this ideal vertical and horizontal. We did this with the apparatus above described, using only the upper disc. The subject simply set this disc until the line upon it appeared to him exactly vertical or horizontal. We also had him set it in a diagonal position 45°, with the vertical or horizontal, speaking of these as "left oblique" or "right oblique," according as the upper end of the line pointed to the left or right. Each set consisted of 20 settings, in which the four positions, vertical, horizontal, "left oblique" and "right oblique," occurred in a chance order.

Observations were made upon ten subjects, eight of whom set each line twenty-five times (five sets) and two of whom set each line fifty times (ten sets). In the following table appear for each subject the number of settings of each position, the resulting average for each position and the average variation of the twenty-five (respectively fifty) records from their mean value.

The last line of the table averages these results for the entire ten subjects:

| Subject. | No. | Vertical. | | Horizontal. | | Right Oblique. | | Left Oblique. | |
|---|---|---|---|---|---|---|---|---|---|
| | Obs. | Setting. | Variation. | Setting. | Variation. | Setting. | Variation. | Setting. | Variation. |
| J. J. | 50 | 89° 26′ | 0° 31.7′ | 179° 29′ | 0° 34.4′ | 42° 02′ | 2° 29.4′ | 134° 58′ | 1° 52.5′ |
| W. D. B. | 50 | 89° 51′ | 0° 45.3′ | 179° 19′ | 0° 30.7′ | 44° 59′ | 2° 41.2′ | 135° 46′ | 2° 29.0′ |
| F. S. | 25 | 90° 40′ | 0° 49.2′ | 180° 16′ | 0° 35.8′ | 42° 49′ | 2° 52.2′ | 138° 41′ | 3° 12.1′ |
| F. E. B. | 25 | 90° 42′ | 0° 40.1′ | 180° 39′ | 0° 54.7′ | 36° 25′ | 4° 04.4′ | 141° 25′ | 3° 58.2′ |
| E. P. S. | 25 | 92° 35′ | 0° 34.2′ | 182° 14′ | 0° 34.1′ | 44° 08′ | 2° 36.7′ | 139° 44′ | 1° 22.3′ |
| J. H. D. | 25 | 90° 26′ | 0° 25.5′ | 179° 52′ | 0° 51.4′ | 41° 16′ | 2° 04.8′ | 139° 20′ | 2° 28.2′ |
| J. H. T. | 25 | 88° 53′ | 0° 36.9 | 178° 52′ | 0° 57.5′ | 38° 40′ | 2° 52.1′ | 137° 40′ | 2° 59.0′ |
| C. M. R. | 25 | 90° 02′ | 0° 31.2′ | 180° 13′ | 0° 36.2′ | 41° 17′ | 4° 07.3′ | 146° 12′ | 3° 37.2′ |
| G. W. M. | 25 | 90° 09′ | 0° 30.4′ | 180° 02′ | 0° 24.1′ | 42° 39′ | 3° 16.6′ | 136° 42′ | 4° 09.0′ |
| E. T. V. | 25 | 89° 40′ | 0° 33.8′ | 180° 13′ | 0° 31.4′ | 33° 41′ | 2° 08.9′ | 141° 29′ | 3° 16.1′ |
| Average. | | 90° 14′ | 0° 35.8′ | 180° 07′ | 0° 39.0′ | 40° 50′ | 2° 55.4′ | 139° 12′ | 2° 55.4′ |

It appears at once that the ideal verticals and horizontals that we carry with us are exceedingly accurate. This is shown not alone by the close approximation to 90° and 180°, but by the very small average variation, less than two-thirds of a degree. The individual variation is also small; 40.8′ for 90° and 36.1′ for 180° amongst the ten subjects. The diagonal positions show a larger and more constant error; the right oblique, if exact, should be at 45°, but, in all cases, it is less than this. The left oblique, if exact, should but be at 135°, but in all cases [in the first line (J. J.) there is an underestimation of 02′] it is overestimated. This means that in both cases the oblique lines were placed too near the position of the horizontal, in the one case (right) by 4° 10′, in the other (left) by 4° 12′. The average variation is also larger than in verticals and horizontals, being nearly 3°. The individual variations for the ten subjects are 2° 42′ and 2° 26′.

No elaborate comment upon these results is necessary. They give evidence of how thoroughly we have been drilled in the perception of rectangular coördinates; and the small variation both of the individual records and of the subjects is especially noteworthy. Our perception of positions midway between the vertical and horizontal is not so accurate nor so constant, the tendency being to approximate them too closely to the horizontal. The error is the same in direction and extent as that for angles of 45° and 135°, when they were reproduced with the standard angles visible (see Fig. 2 above), but is out of relation to the corresponding reproduction of positions of lines.

## A Further Study of Involuntary Movements.

With the assistance of Thomas P. Carter and Edward P. Sherry.

In a previous contribution (this Journal, Vol. IV. pp. 398-407) there was described an apparatus—the automatograph—by which involuntary movements in the direction of the attention could be readily recorded; and typical illustrations were given of such movements, obtained under various conditions. In further study of these movements we attempted to determine the effect of the position of the body upon them, to analyze them with their constituent factors, and to experiment upon certain other points closely related to these.

If the arm be extended to the side of the body, movements of the hand forward are more readily made than movements backward, and movements toward the body more readily than movements away from the body. The hand moves most easily along a circumference of which the shoulder is the centre. The desideratum is a position in which movements in all directions would be equally easy; while this is almost impossible to secure, it may be approximated by extending the hand at an angle of about 45° with the line joining the shoulders and with the elbows bent at an angle of about 120°. The hand thus extended is placed upon the centre of the automatograph—firmly fixed to the table—and a constant position is secured by outlining in chalk the position of the subject's feet upon the floor. In this way the differences in question were reduced, but not eliminated; the average of all the comparable records at our disposal shows half again as extensive a movement towards an object of attention to the front as towards one to the rear, and a third again as much movement towards as away from the body. In some cases, too, the tendency to move forward overbalances the tendency to move towards the object of attention; in such cases we should have, however, a smaller movement to the front than when the object of the attention was to the front. In brief, the difference in the records accompanying the direction of

the attention to the front and to the rear, seldom fails to appear, although it may appear as a difference in the amount of movement instead of in the direction of movement. Fig. 5 shows a case of the former kind, obtained under different but comparable conditions; Figs. 7 and 8 illustrate the more usual result. The fact that the movements recorded serve as an index of the direction of the attention may thus be established independently of the influence of the position of the body; a conclusion corroborated by results to be described presently.

Observation of the subject's movements during an experiment strongly indicated that the result was complex, and originated in several portions of the body; it seemed both general and local. The chief factor in the general movement was referred to a swaying of the body with the feet as a pivot; this swaying of the head we recorded by fixing the recording plate horizontally on the subject's head[1] and suspending above it the glass pencil, held in an adjustable arm, which was firmly fixed to an upright on the table. The device for holding the writing point is the same as that used in the automatograph and is shown in full size in Fig 3. A cork (C) is pierced by the snugly fitting glass tube (T) and within the tube the pointed glass rod (R) moves easily up and down, accommodating itself to all irregularities of surface or movement; a small rubber band (B) is useful in raising the pencil off the record and in preventing it from falling through the tube. For the record nothing is better than the small ground glass drawing-frames that children use. These nicely hold and stretch the glazed paper; they may be stacked without injury to the record, and the frame prevents the pencil from leaving the record. This device may be variously used and may be recommended as the simplest method of recording movements of the kind in question. It is to be noted that when the subject holds the record-plate and the pencil is fixed, there is recorded a movement in opposite direction to that really made.

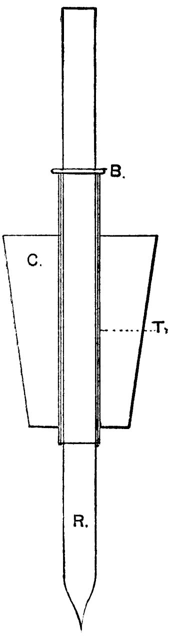

Fig 3. Device for recording movements. The glass rod (R) moves freely up and down in the glass tube (T) held in the cork (C). The rubber band (B) prevents the rod from falling throught the tube.

The movements of the head[2] show the influence of the direction of the attention similarly to those of the hand; indeed the correspondence between the two is considerable and often striking. It appears best when movements of the head and of the hand are recorded at the same time. Fig. 4 may serve to indicate the degree and nature of the correspondence; the head movements are apt to be more extensive and distinctive than those of the hand. This favors the conclusion—to be reinforced by other considerations—that the swaying of the body contributes an important part to the automatograph records.

[1] To fasten this upon the subject's head, a screw eye is fastened to each end of the frame holding the smoked paper; a rubber band is drawn over each arm up to the shoulder; to this band is affixed another rubber band passing through the screw eye and thus securing the frame upon the head. Some soft padding under the frame is desirable.

[2] The records for this study (Figs. 2-12 and 16) have all been taken upon the same individual and thus are as comparable as possible.

We failed to discover any constant tendency to sway in a special direction; movements backward seemed to be as readily made as those forward, and to the right as readily as to the left. When the attention is not directed in any special direction or is directed to a point overhead, an irregular forward and backward as well as lateral swaying results, which is quite different from that accompanying a definite direction of the attention.


Fig. 4. Counting metronome. ⟶ Upper line, movements of head; lower line, of hand on automatograph; time, 45 seconds. The head movements are reversed, but have been again reversed for readier comparison. Figs. 4 to 13 are all obtained upon the same subject. The arrows indicate the direction in which the object attended to was situated.

The most obvious method of eliminating these swaying movements is to experiment with the subject in a sitting position. A typical record of the hand movement on the automatograph with the attention directed to the front appears in I, Fig. 5. The irregu-

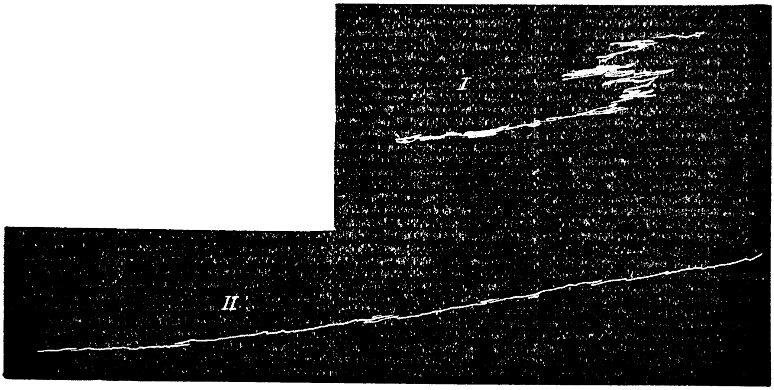


Fig. 5. Counting metronome. Facing ⟶. Automatograph, sitting. I, ⟵; time, 105 seconds. II, ⟶; time, 45 seconds.

lar lateral oscillations have nearly disappeared; the tendency to move along a circumference of which the shoulder is the centre is marked. A more satisfactory method of eliminating the swayings of the body consists of holding the pencil in one hand and the record-plate in the other; in this way pencil and record sway together and thus no record of it is made. Under these conditions we obtain a characteristic type of movement; the several oscillations are small and fine, as appear best when examined with a magnifying glass. Fig. 6 illustrates the type of movement very

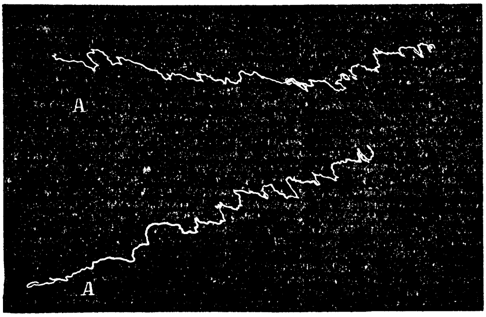


Fig 6. → Counting metronome Right hand holds pencil, left hand holds record; time of each, 90 seconds. Facing →. Upper line, standing; lower line, sitting.

well. It further illustrates that by this method there is no difference, or but a slight one, between the records taken while the subject is standing and while sitting; which is precisely what should be the case if the general movements of the body have been eliminated. In this figure traces are observed of a somewhat regular periodic "curve"; these mark the respirations, and in II, Fig. 7, they are sufficiently distinct and regular to be counted, about twenty to the minute. It is natural that the respiration should appear, because the arm holding the record-plate is rested against the body, and thus records the abdominal movements, though these are apt to be obscured by involuntary movements of the hand holding the pencil. The tendency to move towards the object of attention appears throughout; Fig. 7 further illustrates a movement to the rear as well as to the front, and in Fig. 8 we have an unusually clear indication of readiness with which the direction of the attention may be received. The subject attends to and counts the beats of a metronome, which is in turn carried from one corner of the room to the next; the hand accurately follows the attention, yielding an almost perfect square. In all these tests it is important that a position be chosen in which movements in all directions are equally possible.

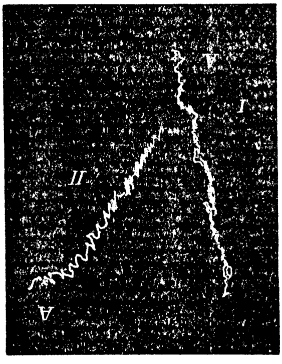


Fig. 7. Thinking of a building. Facing ↑; standing. Right hand holds pencil, left hand holds record; time of each 60 seconds, I, ↑; II, ↓; shows respiration.

As a further test of our analysis of these movements we recorded the movements of the two hands at the same time. This may be done by holding a pencil in each hand over a record-plate placed upon a table, or by holding a record-plate in each hand under a writing point projecting from a table or upright. The difference between these methods is not great; the former method allows of slight finger movements, while the latter does not. The latter is, on the whole, more convenient, because the natural sinking of the hand cannot spoil the record, which might be the case in the other method. The record-plate was placed upon a light board, to which a handle set vertically or horizontally could be attached. Both methods admit of a variety of positions of the arms and hands and dispense with the necessity of maintaining the record-plate level[1]. The results show that the movements of the two hands are very similar indeed; part but not all of this similarity is due to the swaying of the body, which would naturally affect the two sides alike; but there seems also a tendency for the two hands to move together in following the direction of the attention. Fig. 9 illustrates the close similarity of the movements of the two hands. It is important to remember that the records must be made in the same way and the hands be held in the same position. The tendency to move is greater when the hand is held away from than when held close to the body; Fig. 10 illustrates this difference and at the same time shows the correspondence of the form of the movement, notwithstanding the difference in extent.

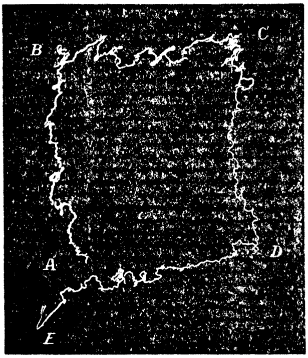

Fig. 8. Counting metronome. Right hand holds pencil, left hand holds record. From A to B, ↑; from B to C, →; from C to D, ↓; from D to E, ←; standing. Each part, 45 seconds.

We have thus illustrated a variety of methods of recording involuntary movements and of analyzing the chief factors contributing to the result. In a measure this separates the mechanical from the psychological portions of the movements and sheds some light upon the positions and methods used in muscle-reading; the additional facilities derivable from the movements of locomotion should not be overlooked.

To this account may be added a few illustrations interesting from various points of view. Involuntary movements are naturally not limited to the horizontal plane; the rod sliding within the tube simply records these alone. We may fix the record-plate in a vertical position against the wall and take the cork between the fingers of the outstretched hand, holding the tube in a slanting position, and thus record vertical movements. This is, however, a fatiguing position, and the fatigue is manifested in a sinking of the hand and arm. This is usually quite rapid and may readily be

[1]The difference between records made with the automatograph and with the device figured in Fig. 1 held in the hand, is mainly one of extent of movement. The automatograph records more finely the tendency to move towards the object of attention as well as the general movements of the body. The illustrations of this article compared with those of the former show the nature of the difference. A further advantage of the automatograph is that it rests the arm.

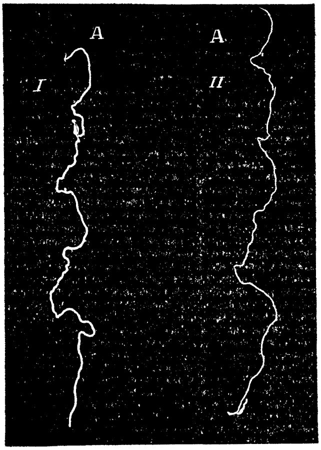


Fig. 9. ↑ Thinking of a building. Standing ↑. ↑ I, left hand; II, right hand; both holding record near the body; time, 35 seconds; records reversed.

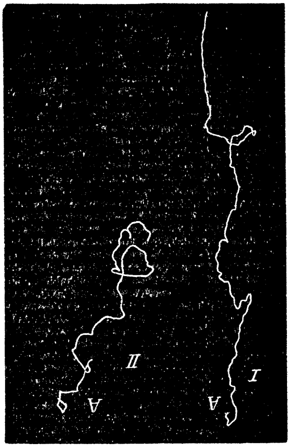


Fig 10. ↑ Thinking of a building. Facing ↓ far out. ↑ I, left hand held extended; II, right hand held close to body; each hand holds record; time, 35 seconds; records reversed.

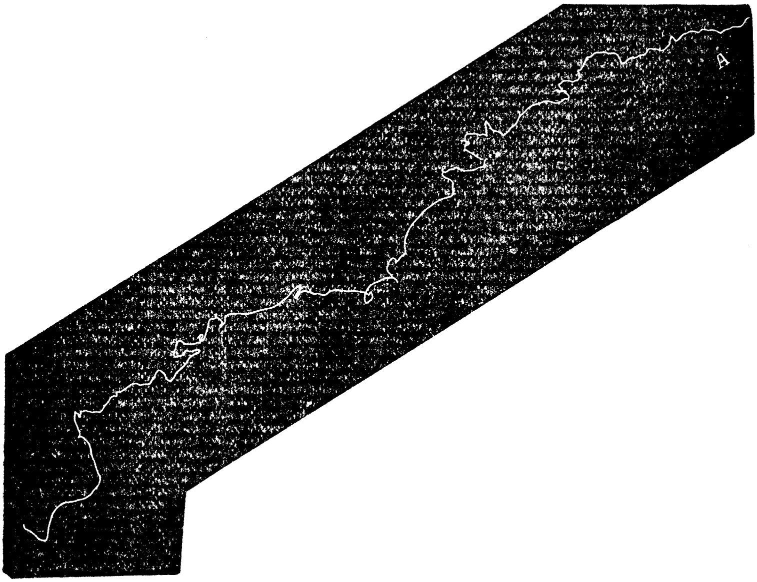


Fig. 11. ← Counting metronome. Record vertical. Facing ←. Time, 20 sec. Pencil held in extended right hand.

recorded; an illustration is given in I, Fig. 12. If in this position the attention is directed forward, we obtain a resultant of the two tendencies, as is shown in the diagonal curve of Fig. 11. Fig. 12

further illustrates an interesting point similar to that illustrated in Fig. 5. In curve I the attention is directed downwards, which quickens, though probably not considerably, the natural tendency for the hand to fall; in curve II the attention is directed to a point overhead, and we observe that this tendency almost exactly balances the effect of the natural fatigue and thus yields this peculiarly interesting result.

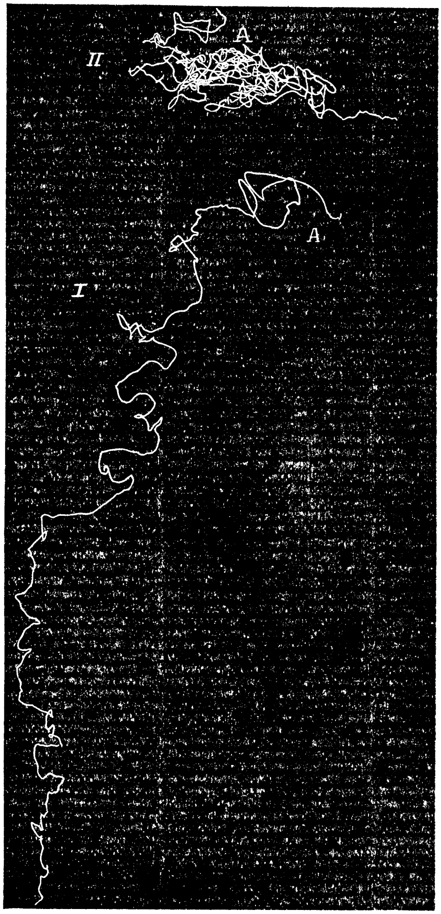


Fig 12. I, ↓ Record-plate vertical. Thinking of one's feet; time, 45 seconds. II, thinking of ↑ a point overhead; time, 45 seconds.

Our attempts to utilize this method for measuring the different degrees of attractiveness of different senses or sense-impressions have not been very successful; and this is mainly due to the great

variability of the result. We have a few illustrations of the difference in question of sufficient interest to reproduce. In Fig. 13

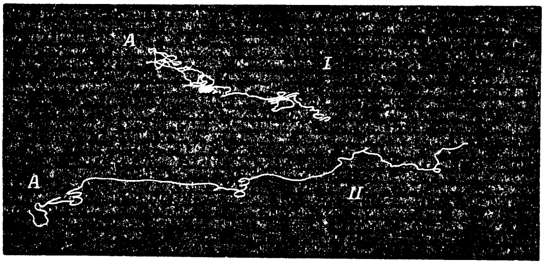


Fig. 13. ⟶ I, counting metronome. Automatograph, facing ⟶. Time, 35 seconds. ⟶ II, counting pendulum. Automatograph, facing ⟶. Time, 25 sec.

curve I represents the movement of the hand while the subject was counting the strokes of a metronome for 35 seconds; the movement is towards the object of attention, but is slight. Curve II represents the movement where a pendulum is substituted for a metronome, a visual for an auditory impression. In this case the usual impression claims the attention more strongly than the auditory; and this corresponds with the subject's analysis of his mental processes. The subject is a noted American novelist and describes himself as a strong visualizer and in general an eye-minded person.

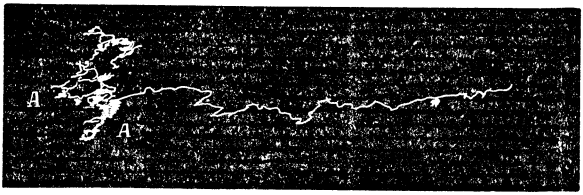


Fig 14. ⟶. Facing ⟶. Hand on automatograph. From A to A ; reading colors; 35 seconds. From A′ on, counting pendulum, 25 seconds.

In Fig. 14 the subject was asked to call the names of patches of colors fixed to the wall opposite him; he did this uncertainly for 35 seconds, his hand moving from A to A′; at this point the counting of a pendulum was substituted for the reading of the colors with a markedly different result, the hand moving directly and rapidly towards the pendulum. Upon examination it proved that the subject's color vision was quite defective, so that the colors did not hold his attention, while the pendulum did. The difference is too marked to be accidental, and is certainly most interesting.

An interesting problem upon which further research is contemplated is the correlation of types of involuntary movements with temperament, age, sex, individual, health and disease, and the like. A few observations were made upon children; they show at once the limited control children possess over their muscles and a similar difficulty in fixing the attention as required. They thus yield an exceedingly irregular result, showing very extensive and coarse

movements, usually towards the object of attention, but with great oscillations. Fig. 15 may serve as a type; in 35 seconds the hand

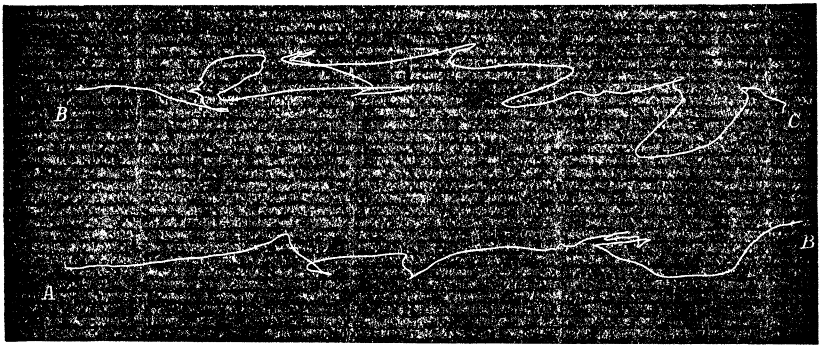


Fig. 15. ⟶ Hand on automatograph. Facing ⟶, counting pendulum. Time, 35 seconds. The record from B′ to C is continuous with that of A to B. The subject, a child of eleven years.

has moved by large skips *seven* inches toward the pendulum, the oscillations of which the child was counting. The difference between this record and those obtained upon adults is striking enough to warrant further study.

Much attention has recently been paid to the subject of automatic writing; in this the subject unconsciously gives indication not of the direction but of the nature and content of his thoughts. We made a few attempts to obtain such upon the automatograph, but entirely without success; we asked the subject to think of a certain letter or simple geometric figure and examine the record for any trace of the outline of this letter or figure. While this method yielded no trace of such a result, it gave us a valuable "control" over the movements in which the attention was directed in a definite direction. In Fig. 16 the subject was thinking of the


**Fig. 16. Thinking of letter O. Pencil in hand; record on table. I, standing; II, sitting.**

letter O; this was not thought of as in any special location and the record likewise fails to show a movement in any special direction. Two records are shown, one while standing and one while sitting, and these will show the difference in the general and local movements in the two cases; the subject is the same as in Figs. 4-12. This adds the corroboration of a negative proof to the general interpretation of the results.

## Further Studies of Classification-Times.

With the assistance of Geo. W. Moorehouse, E. C. Bolton and E. T. Johnston.

This term was applied in the last series of these studies (this Journal, Vol. IV. p. 411) to the time occupied in referring a word or object to its class. The special problem studied related to the time required for answering by the word "noun," "verb" or "adj." (for adjective) when one of ten known nouns, verbs or adjectives was called; i. e., to refer a familiar word to its proper part of speech. This was done when the words were either *nouns or verbs*, *nouns or adjectives*, *verbs or adjectives*, *and nouns*, *verbs or adjectives*. The times were measured by aid of a "mouth-key," in which the release of a bit of wood held between the teeth started or stopped a chronoscope. The natural opening of the teeth in speaking thus served to mark the limits of the time measured. A simple reaction was obtained by answering always with the sound "*eh*" (violent explosive) when the voice of the observer was heard; the action of repeating the word heard was regarded as involving essentially all the steps involved in the classification-time, except that needed for mentally making the classification. In order to bring our former results in direct comparison with those about to be described, we repeated the same set of reactions upon F. E. B. and E. T. J. as were last year made upon the other two observers. The following table gives (in $\sigma = .001$ second) the simple reaction-time; the repetition-time; the classification-time of nouns and verbs, of nouns and adjectives, of verbs and adjectives; the average of these three; and the classification-time of nouns, verbs and adjectives. Each entry represents the average of about 12 sets of 20 reactions each. The lowest line gives the general average of all the results.

| Subject. | Simple Reaction. | Repetition. | Noun-verb. | Noun-adjective. | Verb-adjective. | Average of N.-V., N.-A. and V.-A. | Noun-verb-adjective. | Difference between Simple and Average of N.-V., N.-A. and V.-A. | Difference between Repetition and Average of N.-V., N.-A. and V.-A. | Difference between Average of N.-V., N.-A. and V.-A., and N.-V.-A. |
|---|---|---|---|---|---|---|---|---|---|---|
| J. J. | 190 | 367 | 599 | 595 | 593 | 596 | 667 | 406 | 229 | 71 |
| G.W.M. | 195 | 280 | 628 | 597 | 679 | 635 | 678 | 440 | 355 | 43 |
| F. E. B. | 201 | 364 | 579 | 623 | 604 | 602 | 663 | 401 | 238 | 61 |
| E. T. J. | 179 | 366 | 612 | 619 | 611 | 614 | 684 | 435 | 248 | 70 |
| Aver. | 191 | 344 | 604 | 608 | 622 | 612 | 673 | 421 | 268 | 61 |

From this table we may gather that it takes $191\sigma$ to signal by a vocal reaction that a sound has been heard; $344\sigma$ to repeat a spoken word; the difference between these $153\sigma$ for the processes of distinguishing the sound and calling into play the proper vocal utterance; $612\sigma$ to refer a spoken word to one of two grammatical

classes, there being no difference amongst the three pairs of parts of speech used; 673$\sigma$ or 61$\sigma$ additional to do this when the spoken word may be one of three classes of ten each; and 268$\sigma$ for the mental process of deciding and recalling what the appropriate part of speech is.[1]

Our next step was to substitute for the grammatical classes the three classes of *animals*, parts of the human *body*, and articles of *dress;* as before, ten distinctive and familiar monosyllabic words were chosen as follows: *dog*, *fox*, *hen*, *pig*, *cow*, *bee*, *snake*, *goose*, *goat*, *horse;* *eye*, *ear*, *nose*, *mouth*, *head*, *arm*, *hand*, *foot*, *tooth*, *thumb;* *hat*, *cap*, *coat*, *vest*, *glove*, *shoe*, *boot*, *tie*, *cuff*, *shirt*. We answered "*an*," when the name of an animal was spoken; "*bod*," for parts of the body; and "*dress*," for articles of dress. As before, we classified these words when they were selected from *animals* and parts of the *body*, from *animals* and *dress*, from *body* and *dress*, and from *animal*, *body* and *dress;* and in all respects repeated for these classes what he had done for the grammatical ones. Our results are embodied in the following table, in which each entry is the average of 12 sets of 20 reactions each:

ANIMALS, BODY, DRESS.—*Verbal Series.*

| Subject. | Animal-body. | Animal-dress. | Body-dress. | Average of A.-B., A.-D., B.-D. | Animal-body-dress. | Difference between Simple and Average A.-B., A.-D., B.-D. | Difference between Repetition and Average of A.-B., A.-D., B.-D. | Difference between Average of A.-B., A.-D., B.-D., and A.-B.-D. |
|---|---|---|---|---|---|---|---|---|
| J. J. | 747 | 768 | 693 | 736 | 809 | 546 | 369 | 73 |
| G. W. M. | 567 | 575 | 551 | 564 | 637 | 369 | 284 | 73 |
| F. E. B. | 724 | 709 | 683 | 705 | 783 | 504 | 341 | 78 |
| E. T. J. | 658 | 694 | 673 | 675 | 737 | 496 | 309 | 62 |
| Aver. | 674 | 686 | 650 | 670 | 741 | 479 | 326 | 71 |

Several conclusions drawn from the former table reappear in this one. The three kinds of classifications with two classes take about equal times; the additional time needed to make the same classification when the word may be one of three classes is about the same. Further, the process of classifying words as *animals*, *body* or *dress* is longer than classifying them as parts of speech for three subjects, for J. J. by 140$\sigma$, for F. E. B. by 103$\sigma$, for E. T. J. by 61$\sigma$, but 71$\sigma$ shorter for G. W. M. G. W. M. experienced

[1] It is also noted that our additional research entirely confirms the conclusions drawn in the last study, viz., that the results for J. J. are more typical than the others. The peculiar difficulty of G. W. M. to distinguish verbs from adjectives still affects the results. But the main irregularities in our former results, due to irregularities of practice, do not reappear, owing to the fact that care was taken that the various kinds of reaction be equally distributed throughout the work.

unusual difficulty in naming the parts of speech of words, and found the classification with *animal*, *body*, *dress* somewhat easier than the others; it is, perhaps, fair to regard the average of the other observers 102$\sigma$ (or an increase of 17%) as representing the increased difficulty involved. It is easier upon hearing the word dog, to recall and say that *dog* is a *noun* than that *dog* is an *animal;* which would in turn indicate that we have been better schooled in recognizing the parts of speech of words than in recognizing the more or less natural classes into which the objects denoted fall.

Our three groups of ten words each were chosen with reference to easy pictorial illustration, for our design included the classification of the pictures of these objects as well as of the spoken words. For this purpose pen and ink drawings of the thirty objects were prepared and fastened upon small slips of glass; the drawings were of a uniform size, the extreme outlines being contained within a circle 35 mm. in diameter. A frame was made in which ten of these pictures could be mounted in a carriage and moved along horizontally in back of a shutter such as the photographers use. This shutter consisted of two wings in back of an opening of adjustable size; a pressure on an air bulb withdrew the wings from the opening in the usual way, and in so doing established an electric circuit by which the chronoscope was started. By the aid of a series of spring stops and a weight to move the carriage, we could conveniently and quickly bring any one of the ten pictures behind the opening in the shutter. The reaction was made as before by speaking the appropriate class name with the mouth-key. After each ten reactions the pictures were changed and two series of ten reactions each constituted a set. The following table, arranged similarly to the former tables, gives the results of our "pictorial" series. Each entry under "simple" and "naming" represents the average of 12 sets of 20 observations each; each entry under the other columns of 8 such sets.

ANIMAL, BODY, DRESS.—*Pictorial Series.*

| Subject. | Simple. | Naming. | Animal-body. | Animal-dress. | Body-dress. | Average of Animal-body, Animal-dress, Body-dress. | Animal-body-dress. | Difference between Simple and Average of A.-B., A.-D., and B.-D. | Difference between Average of A.-B., A.-D., and B.-D. and A.-B.-D. |
|---|---|---|---|---|---|---|---|---|---|
| J. J. | 202 | 484 | 524 | 526 | 570 | 540 | 603 | 338 | 63 |
| G. W. M. | 214 | 522 | 547 | 494 | 532 | 526 | 605 | 312 | 79 |
| F. E. B. | 235 | 563 | 563 | 561 | 577 | 567 | 639 | 332 | 72 |
| E. T. J. | 185 | 558 | 545 | 507 | 524 | 526 | 589 | 341 | 63 |
| Aver. | 209 | 532 | 545 | 522 | 551 | 539 | 608 | 330 | 69 |

Again it appears that the classification of a word into any one of the three pairs of classes requires about equal times, and that the

additional time to do this for one of three classes remains the same, about $70\sigma$.

The chief result of a comparison of this with the former table is the fact that it takes *less time to classify a picture than a word;* less time to recall and say that "dog is an animal" when the picture of a dog is shown than when the word is spoken. While the classification in the verbal series (for two classes) requires $670\sigma$, in the pictorial series it requires only $539\sigma$, or $131\sigma$ less; for classification into three classes, $741\sigma$ and $608\sigma$, or $133\sigma$ less. It is fairer to take account of the differences in the simple reaction-time of the verbal and pictorial series, $191\sigma$ and $209\sigma$; and thus the difference in the mental processes of classification is greater by $18\sigma$ than the differences just given.

In the verbal series we found reasons for regarding the time of repeating a spoken word as involving all the processes of classifying the word except the act of recalling the classification (see Vol. IV. pp. 412–413); the pure mental classification time for the grammatical series (two classes) would thus be $268\sigma$, for the verbal animal-body-dress series $326\sigma$. In the pictorial series we were unable to devise any means of making this elimination, and so cannot say how much of the difference between the simple reaction and the entire classification-time, $330\sigma$, is taken up by the process of recognizing and indicating the recognition of the picture, how much by the recalling of its class name. There are strong reasons for believing that very much the greater portion of the $330\sigma$ is taken up in the mental classification process.

It is further of interest to compare the process of classifying with that of naming. Is it a more complicated process, upon seeing the picture of a dog, to say "dog" or to say "animal"? Do we first recognize the lines as representing a dog, and then decide that a dog is an animal, or do we at once recognize the drawing as that of an animal? We are able to give but an imperfect answer to this question. For J. J. it is easier to name than to classify, and the time is shorter by $56\sigma$; for G. W. M. and F. E. B. there is practically no difference; for E. T. J. naming requires $32\sigma$ longer. The inferences from these results are that the two processes are about of equal complexity, that it is unlikely that the specific recognition of the class to which the object belongs includes the recognition of the individual object, and that the processes may be different in different persons.

We have already noted that the three pairs of distinctions of classes are of equal difficulty; it is further of interest to ascertain whether it is easier to pronounce a word a noun, verb or adjective; a name, a picture, that of an animal, of a part of the body, or of an article of dress. The following table gives the data for this decision, and by noting the numbers in bold type we see that on the average of all cases in which a word was pronounced a noun, the time required was $616\sigma$, for verbs $627\sigma$, for adjectives $651\sigma$; for animals $695\sigma$, for body $690\sigma$, for dress $698\sigma$; for pictures of animals $555\sigma$, for body $568\sigma$, for dress $564\sigma$. In each class the three types of classification are thus of practically equal difficulty. The increase of time needed to pronounce a word an adjective above that needed to pronounce it a noun is of note, however, amounting to $35\sigma$ in the average and appearing in each of the four individuals. The individual records agree well with their average. The table will doubtless be clear without further comment:

| Subject. | Noun. | | | | Verb. | | | | Adjective. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | N.-A. | N.-V. | N.-V.-A. | Aver. | N.-V. | V.-A. | N.-V.-A. | Aver. | N.-A. | V.-A. | N.-V.-A. | Aver. |
| J. J. | 604 | 594 | 631 | 610 | 594 | 568 | 651 | 604 | 596 | 618 | 698 | 637 |
| G. W. M. | 620 | 568 | 657 | 615 | 636 | 669 | 675 | 660 | 627 | 689 | 701 | 672 |
| F. E. B. | 594 | 626 | 610 | 610 | 564 | 604 | 652 | 607 | 620 | 604 | 728 | 651 |
| E. T. J. | 609 | 612 | 665 | 629 | 615 | 613 | 683 | 637 | 626 | 607 | 704 | 646 |
| Average. | 607 | 600 | 641 | **616** | 602 | 613 | 605 | **627** | 617 | 629 | 708 | **651** |

| Subject. | Animal. | | | | Body. | | | | Dress. (Words.) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A.-B. | A.-D. | A.-B.-D. | Aver. | A.-B. | B.-D. | A.-B.-D. | Aver. | A.-D. | B.-D. | A.-B.-D. | Aver. |
| J. J. | 744 | 764 | 806 | 771 | 750 | 687 | 806 | 748 | 772 | 701 | 814 | 762 |
| G. W. M. | 566 | 547 | 628 | 580 | 568 | 549 | 632 | 583 | 603 | 555 | 651 | 603 |
| F. E. B. | 723 | 739 | 743 | 735 | 725 | 683 | 804 | 737 | 679 | 683 | 803 | 722 |
| E. T. J. | 657 | 680 | 742 | 693 | 659 | 667 | 746 | 691 | 708 | 679 | 723 | 703 |
| Average. | 674 | 682 | 730 | **695** | 675 | 647 | 747 | **690** | 690 | 655 | 748 | **698** |

| Subject. | Animal. | | | | Body. | | | | Dress. (Picture.) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A.-B. | A.-D. | A.-B.-D. | Aver. | A.-B. | B.-D. | A.-B.-D. | Aver. | A.-D. | B.-D. | A.-B.-D. | Aver. |
| J. J. | 521 | 517 | 581 | 540 | 527 | 569 | 619 | 572 | 535 | 571 | 610 | 572 |
| G. W. M. | 557 | 501 | 592 | 550 | 537 | 514 | 620 | 564 | 485 | 550 | 592 | 542 |
| F. E. B. | 561 | 562 | 622 | 582 | 567 | 576 | 635 | 593 | 560 | 578 | 661 | 600 |
| E. T. J. | 554 | 495 | 592 | 547 | 536 | 528 | 589 | 551 | 519 | 522 | 587 | 543 |
| Average. | 548 | 519 | 597 | 555 | 542 | 547 | 616 | 568 | 525 | 555 | 612 | 564 |

It is also true that it takes practically the same time to name a picture of an animal, a part of the body, or an article of dress. This appears in the following table:

NAMING PICTURES OF OBJECTS.

| Subject. | Animal. | Body. | Dress. | Average. |
|---|---|---|---|---|
| J. J. | 488 | 469 | 494 | 484 |
| G. W. M. | 530 | 513 | 524 | 522 |
| F. E. B. | 573 | 569 | 546 | 563 |
| E. T. J. | 557 | 558 | 558 | 558 |
| Average. | 537 | 531 | 527 | 532 |

The final average ($532\sigma$) represents the average of 960 reactions, animals being named 340 times, parts of the body 266 times, and articles of dress 354 times. We also append a statement of the number of errors, i. e., in which a word or picture was referred to a class not its own. Our data for nouns, verbs and adjectives relate only to F. E. B. and E. T. J.; for these the numbers of errors in percentage of observations taken are as follows:

| SUBJECT. | N.-A. | N.-V. | V.-A. | N.-V.-A. | Average. |
|---|---|---|---|---|---|
| F. E. B. | 1.67 | 6.17 | 6.17 | 2.92 | 4.27 |
| E. T. J. | 2.92 | 1.67 | 1.67 | 1.25 | 1.88 |
| Average. | 2.30 | 3.92 | 3.92 | 2.09 | 3.08 |

For the verbal series the errors in percentage are:

| SUBJECT. | A.-B. | A.-D. | B.-D. | A.-B.-D | Average. |
|---|---|---|---|---|---|
| J. J. | 7.92 | 5.42 | 3.33 | 5.83 | 5.77 |
| G. W. M. | 8.92 | 12.92 | 12.92 | 14.54 | 12.29 |
| F. E. B. | 5.00 | 2.92 | 5.42 | 2.50 | 3.96 |
| E. T. J. | 4.58 | 6.17 | 5.00 | 1.45 | 4.38 |
| Average. | 6.61 | 6.86 | 6.67 | 6.08 | 6.60 |

Average of F. E. B. and E. T. J., 4.17.

For the pictorial series the errors in percentage are:

| SUBJECT. | A.-B. | A.-D. | B.-D. | A.-B.-D. | Average. |
|---|---|---|---|---|---|
| J. J. | 1.25 | 3.75 | 0.63 | 0.67 | 1.56 |
| G. W. M. | 10.00 | 0.63 | 3.75 | 8.12 | 5.62 |
| F. E. B. | 0.63 | 1.25 | 1.87 | 1.25 | 1.25 |
| E. T. J. | 0.63 | 0.63 | 1.87 | 2.50 | 1.41 |
| Average. | 3.13 | 1.57 | 2.03 | 3.12 | 2.46 |

Average of F. E. B. and E. T. J., 1.33.

It appears that the percentage of error is smallest for the pictorial series, largest for the verbal series, and intermediate for the grammatical series. The individual difference that should be noted is the large number of errors of G. W. M., which is undoubtedly related to the shortness and mode of his reactions. The order of the four subjects regarding their liability to error is the same in the

verbal and pictorial series. It is also of interest to inquire whether the average time of these erroneous reactions is markedly different from the time of the correct ones. In the following table the average time of the erroneous reactions is given in percentage of the corresponding average correct reactions; and the results show, on the whole, no appreciable difference between the two. In the erroneous reactions there is probably a greater variation than in the correct ones:

| | J. J. | G. W. M. | F. E. B. | E. T. J. | Average. |
|---|---|---|---|---|---|
| Grammatical Series. | . . . . . | . . . . . | 94.8 | 119.5 | 107.1 |
| Verbal Series | 108.6 | 97.6 | 99.9 | 103.1 | 102.3 |
| Pictorial Series. | 97.2 | 105.3 | 107.0 | 97.2 | 101.7 |

## On the Perception of Simultaneous Sense-Impressions.

With the assistance of George W. Moorehouse, Fellow in Psychology.

The error in indicating with which one of a series of visual impressions an auditory or other impression seems simultaneous, was first noted by Wundt. He studied it by having an index rotate in front of a graduated disc at a constant rate, or again by having it oscillate with a pendular movement, and noting to what stroke of the disc the hand seemed to be pointing when a bell sounded. The actual moment of the sound was determined by the observer by moving the pendulum slowly across the disc and the error in time was then calculated by mathematical formula. With this apparatus Wundt established that it takes many separate judgments before one is ready to make one's decision; that the error is very variable; that the error for almost all the rates of movement used is negative, i. e., the time at which the bell is said to have rung precedes the time of its actual ringing; that this error decreases as the speed increases, until it becomes positive; that the error increases in the accelerating portion of a pendular movement, and decreases in the portion of diminishing velocity; that the error with the constant motion disappears when one division[1] of the disc corresponded to $\frac{1}{36}$ of a second ($=28\sigma$) and the interval between successive sounds of the bell is one second; and finally the very important fact that the determination was considerably under the control of the will of the observer, and was influenced by the direction and nature of the attention.

Wundt also experimented with more than two simultaneous impressions, but his results on this point need not now be considered.

Tschisch (Wundt's Studies II. 603-634) has contributed an elaborate research, working with the same apparatus, but his main results are concerned with the determination of the error with several simultaneous impressions. Reference to his results will be made later on. Both Tschisch and Wundt connect with their results an elaborate theoretical interpretation.

[1] This division refers to the smallest portion of the divided circle taken into account in the subject's judgment; Wundt's apparatus as figured has a mark for every two degrees, but it is to be inferred that he judged only to the nearest ten degrees. The importance of this point is the subject of discussion below.

For several reasons a reinvestigation of the fundamental factors of these interesting phenomena seemed desirable; the accepted interpretation of the error as the time needed for the reception and elaboration of the perception (*Complication einer Vorstellung*[1]) seemed questionable; the dependence of the error upon the apparatus as well as upon the mode of judgment seemed not to have been sufficiently regarded. The phenomenon, when reduced to its simplest terms, may be thus described: There is a series of sense-impressions following one another in a recognizable order and the members of which are distinguished from one another both in time and by some other characteristic; a disparate and momentary sense-impression is interposed at some moment unknown to the subject, and he must determine with which one of the series of impressions the disparate impression seemed to coincide. In order to take note of small errors, it is necessary that the successive members of the series of sense-impressions be rapidly distinguished; and sight and hearing alone, therefore, are available for this purpose. It is true that we can distinguish both these and other sense-impressions by the artificial device of counting, but this process is too slow and absorbs too great a share of the attention to be here available. Sight is decidedly the preferable sense by reason of its superior power of taking in a large range of impressions at once; and in many ways the most convenient visual impressions are the divisions of a divided circle. The place of a given mark in the circle is readily determined when each fifth or tenth mark is differentiated from the others; the circle used by Wundt has a short stroke for each two degrees and a larger stroke for each ten degrees, and this division we have used in our experiments. A point travels along this visual scale, and for the interposed impression the stroke of a bell or an electric shock on the finger is most convenient. Our problem then is simply this: Where upon the divided circle was the moving point when the bell sounded or the shock was felt? The most important factor in this decision is obviously the accuracy with which the subject is required to decide; i. e., whether he is to determine the point when the bell sounded to the nearest ten, five, two or one degree; this is the one point that must be determined before the observations can proceed, and indeed must be considered in the preparation of the divided circle. And yet it is a surprising fact that this is the one point upon which former observers have been most reticent; one can only infer it, and that not too certainly from the apparatus used. Wundt, in his observations with an index revolving at a constant rate, judged to the nearest ten degrees; Tschisch apparently judged more accurately, and if by a division (Theilstrich) he means a division of Wundt's apparatus as figured, he judged to the nearest two degrees. The accuracy of the judgment with a given rate of movement is dependent upon the size of the divisions; after a certain velocity has been reached, we can no longer distinguish the several positions of the moving point. The larger the disc, the higher the speed as measured by the time of one revolution at which an interval of a given number, say two degrees, may be distinguished. Wundt's disc for constant movement was but slightly over six inches in diameter, and thus it is clear why he could judge only to the nearest ten degrees; in the apparatus of Wundt used by Tschisch, the disc is about 8⅝ inches in diameter.

[1] Tschisch has elaborated and Wundt has endorsed this interpretation as applied to more than two simultaneous impressions; and a table is given indicating the time of the several higher kinds of reception and fusion of perceptions (Tschisch p. 633; Wundt, 3rd Ed. II, p. 341).

To obtain a greater range of velocity of movement we used a much larger disc, 22 inches in diameter, divided by short strokes into two degrees, and by a longer one for every ten degrees. We judged not only to the nearest stroke, but also whether the point stood on or between two strokes when the bell sounded, i. e., we *judged to the nearest degree.* The distance on our disc between two strokes (two degrees) was 9.75 mm., or about $\frac{3}{8}$ of an inch; in Wundt's disc this distance was only 3.84 mm., about $\frac{3}{20}$ of an inch. Furthermore, to secure ease of reading, the markings were plain and bold, in black ink upon white card-board, and the index was blackened and tapered to a readily visible point.

In almost all previous determinations, the index moved with a pendular movement; while it is interesting to observe the effect of the change of velocity upon the error of judgment, it is certainly important to have determined the error for a constant rate as a standard of comparison; and to regard the rate at the base of a pendular oscillation as equivalent for this purpose to a constant rate is not free from objection. In all our observations, the index traveled over the disc at a constant rate.

This disc was glued to a board, 22 inches square, and the whole mounted in a vertical position; through a hole in the centre of the disc, the axle bearing the index projected and this index could be set in any position and then fastened by a thumb-screw. The mechanism by which the index was rotated was the clock-work of a clinostat. This apparatus was admirably adapted to our purpose and admitted of a great range of velocity. On the axle, behind the disc, was fastened a small wire, the end of which just dipped into a mercury drop, and thus in each revolution established an electric circuit. By this connection, a bell could be struck or an electric shock given to the finger, and by the setting of the index the point at which this occurred was charged. Moreover, a switch in each circuit enabled the observer to introduce the sound or the shock at any desired moment; this is important, as no judgment should be made until the clock-work has obtained its full and constant rate. The subject sat at a convenient distance before the disc, the latter concealing from his view all the mechanism by which the index was rotated as well as the bell, and called out the positions at which the sound or shock seemed to come.

A further point in the method of observation is of importance. The judgment in which the subject has any confidence is formed only after several observations, the point at which the impression was interposed shifting with each observation. There are two natural methods of recording the error; the one is to take the *average of all* the observations with a given setting of the interposed impression; the other is to ask the subject to decide upon *one judgment as the final* one, and to measure the error by this, recording, however, the several observations as well. After a trial of both, we adopted the latter plan as the better. The point at which the interposed impression really occurred was readily determined by slowly moving the index (by turning with the finger one of the fine wheels of the clock-work) until the bell sounded or the shock was felt. In order to have the sound or shock as brief as possible, the mercury cup was made in the form of a narrow slit, through which the point of the wire could be made to pass at any desired angle, and to prevent the sound of the bell from continuing after it was struck, the bell was loaded with drops of wax. In some cases we found it more convenient to use a spring wire instead of the mercury drop. The rate of the index was determined by timing to the nearest second three, five or ten revolutions, according to the

rate, and the result was expressed uniformly *in the $\sigma$=.001 of a second required for the point of the index to travel over one degree of the circumference.*

The results we have thus far accumulated are in every sense provisional; the number of observations is not adequate and they are offered at the present time simply because of the interest in the methods by which they have been obtained and their explicit disagreements with previous results.

We calculate from the tables of Tschisch, that with the index moving at such a rate that 1° was passed over in 3.07 $\sigma$, the error for sound was a negative one of 64.8 $\sigma$, for touch 64.8 $\sigma$, for electric shock 72.2 $\sigma$; with a faster rate of 1° in 2.41 $\sigma$, these errors were 44.1 $\sigma$, 44.1 $\sigma$ and 39.9 $\sigma$; and with a still faster rate of 1° in 1.7 $\sigma$, these errors were 20.3 $\sigma$, 20.3 $\sigma$ and 20.3 $\sigma$. In all cases these are the errors at the base of the pendular movement, when the acceleration is zero; and by a negative error is meant an error in judging the interposed impression as occurring in advance of its actual occurrence. The sound was that of a bell, the touch a tap of a hammer upon the frontal surface of the last joint of the forefinger and the third kind of stimulus was an electric shock, presumably upon the finger. Tschisch does not describe his manner of recording the judgments, whether he averaged all the observations or accepted a final judgment with each test; in what way he combined errors of opposite direction and the like. The chief characteristics of his result are the large size of the errors; the decrease of the error with an increase of speed and that, too, within small range; the constancy of this error with different kinds of interposed stimuli, and the negative character of the error throughout.

In our observations, the individual variation of the results is so very great that it seems somewhat strained to attach any importance to the general average. These variations are so great that in all the observations with any one rate of speed, observations with positive as well as negative errors occur. Furthermore, within the range of velocities studied by Tschisch, we can distinguish no constant tendencies at all, and within the very much larger range of velocities at our disposal nothing that could be dignified by the name of law appears. Taking all our observations together we find for the sound:

| | | | | |
|---|---|---|---|---|
| Rate, | 1° in 2 to 4 $\sigma$. | 1° in 4 to 6 $\sigma$. | 1° in 6 to 9 $\sigma$. | 1° in 9 $\sigma$ or more. |
| Error, | —10.6 $\sigma$ | +7.0 $\sigma$ | +5.9 $\sigma$ | +1.8 $\sigma$ |

These numbers are based upon 120 observations in all.

With the electric shock as the stimulus the results are:

| | | | | |
|---|---|---|---|---|
| Rate, | 1°=2 to 4 $\sigma$. | 1°=4 to 6 $\sigma$. | 1°=6 to 8 $\sigma$. | 1°=8 $\sigma$ or more. |
| Error, | —15.8 $\sigma$ | —3.5 $\sigma$ | —6.4 $\sigma$ | —45.1 $\sigma$ |

These results are based upon 130 observations in all.

These results, though entirely provisional and without much significance, owing to the great individual variations, yet are opposed to all the four main results of Tschisch's experiments. What this opposition means, it would be premature to say. But three points, further, need be noticed: (1) the relative constancy of the results when calculated without regard to their positive or negative characters; (2) the difference of individual observers in these observations; (3) the tendency of the several individual judgments in a single observation. With regard to (1) it is only necessary to indicate this fact: this error is not far from 30 $\sigma$ for the sound, and from 40 $\sigma$ for the electric shock, independently of rate. (2) We have tested a sufficient number of individuals to make great differences in the size and direction of the error, but not enough to describe them in quantitative terms. With regard to (3) we can only say that with

some individuals the index nearly always moves to a later point of the disc with successive sounds of the bell or shocks on the finger.

We did not confine our studies to this method of observation, but devised several others, to the description of which we may now turn.

We arranged a method by which a series of auditory impressions could be substituted for the visual ones. This arrangement requires the services of three persons; in one room there is the subject with his finger on the "shock key," listening to the reading of the observer; the observer, in reading, speaks into the mouth of the transmitter of a telephone, and at the other end of the telephone in another room sits the recorder; there is, further, an arrangement by which, either automatically or at the desire of the recorder, the shock may be given and the telephone circuit broken. The subject notes at what word he was listening when the shock came and the recorder records the last word heard before his telephone was cut out of the circuit. A simpler mode of observation consisted in connecting the shock circuit with the telephone circuit and noting between what words the slight sounds accompanying the making and breaking of the shock circuit came. We then measure the rate of speaking and calculate the error in $\sigma$. In reading from a book, the subject does not know what is coming; but in observing the movements of an index before a disc, the sequence of impressions is foreknown. We can secure the latter conditions for hearing by counting, or by speaking the alphabet. A. further variation consists in having the subject himself count aloud or read aloud from a printed page; but this is not so serviceable as the other form of experiment. The result of all our experiments with the auditory series may thus be expressed: The error in indicating the place of a shock in an auditory series is less than one of the smallest units of time (the time needed to speak one word or one syllable) that we could take into account in the observations. The quickest sense-impressions that one can follow by ear is counting from one to ten repeatedly, but this can hardly be done more rapidly than seven per second; our result then simply shows that the error is rarely as large as one-seventh of a second$=143\ \sigma$. It is further to be noted that the recorder's method of noting the error is not as objective as is to be desired. In noting the place of the two clicks in the auditory series, there is some though less opportunity for the same error of time-location as in placing the position of a shock in the same series. The experimentation is difficult, the results indicating that the ear cannot differentiate and locate the sequence of impressions with sufficient rapidity to permit of the detection of the error under consideration.

As a further contribution to the influence of the apparatus and mode of judgment upon these errors of location in time, we altered one of the most important conditions of all former experiments: instead of having the disc stand still and the index moved, the reverse was done. This was accomplished as follows: Upon a revolving drum was fastened a sheet of paper with various lines of letters, words and numbers written upon it with a type-writer; the drum was in a horizontal position, but to bring the letters in a vertical position and to have them pass across the field of vision from right to left (and thus be read from left to right) as well as to have but one line, or rather as much of one line as one could see, in sight at any one time, two mirrors were appropriately placed at the end of a shallow box, through which the subject read. A fine thread placed in front of one of these mirrors served as an index, the subject judging what letter or number was opposite the thread when

the bell sounded or his finger received the shock. As before, the drum moves at a constant rate, and a final judgment is recorded after several individual observations. We judged always to the nearest letter or number (there were 135 in a line), and in some instances we attempted judgments between or upon letters or spaces, that is, to the nearest half letter. It will be seen that a letter corresponds to $2\frac{2}{3}^\circ$ or a half-letter to $1\frac{1}{3}^\circ$. We used 12 different lines; lines I. and II. were continuous words from a story for children; line III., a series of detached monosyllabic words; line IV., miscellaneous numbers between 20 and 100; line V., the numbers in regular order from 20 on; line VI., the numbers advancing by 7 from 12 to 100, 11 to 100, 13 to 100; line VII., letters of the alphabet in chance order; line VIII., a line of verse; line IX., of prose; line X., the same line of prose, but in reversed order; line XI., a line of German; line XII., a scale of short uniform marks with every fifth mark heavier and numbered to correspond exactly with the divided circle with the rotating index.

Let us consider line XII. first, as that allows of most direct comparison with former results. With this method of judging a much slower rate is necessary; the circle is much smaller, about 4 inches in diameter; it is more difficult to read the lines while in motion, and a smaller portion of the circle is visible at any one time.

With the bell our results are :

| | | | | |
|---|---|---|---|---|
| Rate, | $1^\circ$ in 20 to 24 $\sigma$. | $1^\circ$ in 24 to 28 $\sigma$. | $1^\circ$ in 38 to 42 $\sigma$. | 75 |
| Error, | —27.6 $\sigma$ | —20.7 $\sigma$ | —28.8 $\sigma$ | observations. |

With the electric shock, the results are :

| | | | | |
|---|---|---|---|---|
| Rate, | $1^\circ$ in 20 to 24 $\sigma$. | $1^\circ$ in 24 to 28 $\sigma$. | $1^\circ$ in 32 to 50 $\sigma$. | 60 |
| Error, | —50.7 $\sigma$ | —23.2 $\sigma$ | —54.4 $\sigma$ | observations. |

The great variability of the results is again a striking factor, though they are almost uniformly negative. There is no definite connection suggested between rate and error, and the error is different with the bell and with the shock.

The other eleven lines were arranged to furnish material for the study of the effect of the different kinds of visual series upon the error. Most of the observations were, therefore, made with one rate of speed, about $1^\circ$ in 20 $\sigma$. If we divide the lines into those containing continuous words, I., II., VIII., IX., XI., those containing detached words or letters, III., VII., X., and those containing numbers, we find as the general average error of the first set $+2.2\sigma$, of the second $+7.0\ \sigma$, of the third, $0.0\ \sigma$. There is no difference of note between the results of bell and shock. The small error and great variability of the results are again the marked characteristics; the averages have thus little significance. But one further result is worthy of notice. If, instead of recording simply the final judgment, we record all the individual answers and enter their average as a result, then the error seems to be larger; we can make such a comparison for the "shock-stimuli," though the two sets of results were not taken at the same rate (the "average" judgments being at about half as rapid a rate as the "final" ones). In the first case the average error is 4.1 $\sigma$ and in the second case 35.7 $\sigma$. Attention is again directed to the provisional character of the results throughout, and to the fact that our main objects are the analysis of the factors involved in these observations, the indication of the dependence of the results upon method, apparatus and mode of judgment, and the recording of the absence of agreement of our provisional results with those obtained by other observers.

## THE PSYCHO-PHYSIC SERIES APPLIED TO LIFTED WEIGHTS.

With the assistance of WALTER D. BROWN.

The method of the psycho-physic series presents to the subject a range of sensation intensities—usually successively one at a time—and requires him to assign each sense-impression to one of a given number of divisions or magnitudes. By this method we may compare the assigned magnitudes of the stars with their photometric intensities and determine whether the subjectively equal different magnitudes (arithmetical series) correspond to a similar objective series or—as the psycho-physic law demands—to a series of equal ratios (geometrical series). In former contributions (Vol. I. pp. 112-127, Vol. III. pp. 44-79, Vol. IV. pp. 213-219) the close correspondence of estimated star-magnitudes with the law was shown, and the method applied with varying results to sensations of visual and tactual expression, to the time-sense and to the motor-sense. A comparison of all the results suggested the generalization that the law probably holds of sensations that are appreciated *en masse*, without conceiving them as divided into units, on a general unanalyzed impression.

Our present study applies this method to the sensations obtained in lifting weights in the palm of the hand, and this includes the sense of muscular contraction as well as the pressure-sense of the palm. Sixty weights were prepared, the lightest weighing 12 grms. and the heaviest 795 grms.; the intermediate weights corresponded to the average of an arithmetical and geometrical series inserted between these limits; in this way the selection of weights, while from the subject's point of view essentially a matter of chance, favored one result no more than the other. A set consisted of 60 observations, each weight being assigned once to a magnitude. The subject, without distinctly seeing the weight, lifted it up and down in his right hand and assigned it according to his sensations, to one of six magnitudes or classes. The lightest weights were grouped as Class I., the heaviest VI. The order of the weights was determined by chance. The weights were made by packing cylindrical boxes $3\frac{1}{8}''$ high and $1\frac{5}{16}''$ in diameter with leaden discs cut to fit the inside of the box and supplemented by felt discs, cotton and shot; the weight was equally distributed throughout the box and were all alike in appearance, being marked by a letter on the bottom. Two sets (120 judgments) were taken upon seven subjects and four sets (240 judgments) upon three subjects. The average weight in grammes and the results of all the weights assigned to each of the six magnitudes or compartments I., II., III., IV., V. and VI. by each of the new subjects is given in the following table, the last line giving the average of all:

| Subject | No. | I | No. | II | No. | III | No. | IV | No. | V | No. | VI | No. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| J. H. T. | 120 | 39.4 | 23 | 103.2 | 23 | 175.4 | 14 | 249.2 | 14 | 370.1 | 18 | 619.4 | 28 |
| F. S. | 120 | 43.9 | 23 | 92.3 | 23 | 178.9 | 16 | 257.2 | 14 | 400.8 | 21 | 640.4 | 23 |
| J. H. D. | 120 | 25.8 | 13 | 71.2 | 17 | 146.0 | 28 | 286.0 | 22 | 438.5 | 20 | 664.3 | 20 |
| F. E. B. | 120 | 27.3 | 14 | 74.6 | 19 | 131.6 | 18 | 240.4 | 24 | 391.8 | 21 | 640.2 | 24 |
| E. P. S. | 120 | 23.6 | 19 | 91.1 | 20 | 181.5 | 28 | 323.4 | 17 | 461.7 | 19 | 696.2 | 17 |
| C. M. R. | 120 | 34.5 | 20 | 97.6 | 23 | 170.7 | 18 | 285.2 | 23 | 433.9 | 15 | 660.0 | 21 |
| E. T. J. | 120 | 40.5 | 26 | 127.8 | 29 | 239.3 | 20 | 369.2 | 16 | 526.2 | 14 | 676.1 | 15 |
| G. W. M. | 240 | 27.0 | 30 | 58.0 | 41 | 148.2 | 39 | 232.6 | 36 | 370.2 | 45 | 624.3 | 49 |
| W. D. B. | 240 | 29.5 | 30 | 75.8 | 34 | 145.1 | 46 | 255.8 | 44 | 408.9 | 40 | 643.8 | 46 |
| J. J. | 240 | 28.2 | 30 | 76.2 | 39 | 151.4 | 47 | 266.1 | 45 | 422.6 | 28 | 620.1 | 51 |
| Average | 1560 | 32.0 | 228 | 86.8 | 268 | 166.8 | 274 | 276.5 | 255 | 422.5 | 241 | 648.5 | 294 |

In the next table are given for each subject, in the upper line, the successive *differences*; in the lower line, the successive *ratios* between the average weights of neighboring magnitudes; in the last three columns are found the averages of these differences and of these ratios, the average deviation of the several differences and of the ratios from their mean (expressed in percentage), and the ratios of these percentages of deviation to one another. In the lowest lines of the table, similar results are given for the general average of all.

| Subject | I-II | II-III | III-IV | IV-V | V-VI | Average | Average Deviation | Ratio |
|---|---|---|---|---|---|---|---|---|
| J. H. T. | 63.8 | 72.2 | 73.8 | 120.9 | 249.3 | 116.0 | 47.7% | 1: 2.51 |
| | 2.62 | 1.70 | 1.42 | 1.49 | 1.68 | 1.79 | 19.0% | |
| F. S. | 48.4 | 86.6 | 78.3 | 143.6 | 239.6 | 119.3 | 48.4% | 1: 3.56 |
| | 2.10 | 1.94 | 1.44 | 1.56 | 1.60 | 1.73 | 13.6% | |
| J. H. D. | 45.4 | 74.8 | 140.0 | 152.5 | 225.8 | 127.7 | 42.4% | 1: 2.37 |
| | 2.76 | 2.05 | 1.96 | 1.53 | 1.52 | 1.96 | 17.9% | |
| F. E. B. | 47.3 | 57.0 | 108.8 | 151.4 | 248.4 | 122.5 | 50.4% | 1: 2.77 |
| | 2.73 | 1.77 | 1.82 | 1.63 | 1.63 | 1.97 | 18.2% | |
| E. P. S. | 67.5 | 90.4 | 141.9 | 138.3 | 234.5 | 134.5 | 35.7% | 1: 1.89 |
| | 3.90 | 1.99 | 1.77 | 1.43 | 1.51 | 2.12 | 18.9% | |
| C. M. R. | 63.1 | 73.1 | 114.5 | 148.7 | 226.1 | 125.1 | 39.8% | 1: 1.93 |
| | 2.82 | 1.75 | 1.67 | 1.52 | 1.52 | 1.85 | 20.6% | |
| E. T. J. | 87.3 | 111.5 | 129.9 | 157.0 | 149.9 | 127.1 | 17.4% | 1: 0.62 |
| | 3.15 | 1.87 | 1.54 | 1.43 | 1.28 | 1.85 | 28.3% | |
| G. W. M. | 31.0 | 90.2 | 84.4 | 137.6 | 254.1 | 119.5 | 53.8% | 1: 2.97 |
| | 2.15 | 2.55 | 1.60 | 1.59 | 1.69 | 1.91 | 18.1% | |
| W. D. B. | 46.3 | 69.3 | 110.7 | 153.1 | 234.9 | 122.9 | 46.3% | 1: 2.86 |
| | 2.57 | 1.91 | 1.66 | 1.60 | 1.57 | 1.86 | 16.2% | |
| J. J. | 48.0 | 75.2 | 114.7 | 156.5 | 197.5 | 118.3 | 39.7% | 1: 2.17 |
| | 2.66 | 1.99 | 1.76 | 1.59 | 1.47 | 1.91 | 18.3% | |
| Average | 54.8 | 80.0 | 109.7 | 146.0 | 226.0 | 123.3 | 40.6% | 1: 2.04 |
| | 2.71 | 1.92 | 1.65 | 1.53 | 1.53 | 1.87 | 19.9% | |

The general result is indicated in the last entry in the column of ratios; *the approximation to a geometrical series is, on the average, twice as close as to an arithmetical series; to this extent the psychophysic law is followed.*

Passing to the individual results, it is seen that all the individuals, with one exception (E. T. J.[1]), favor the geometric series, and of these nine, six approximate it more closely than the general result. If we omit the one divergent record, the general deviation from an arithmetical series becomes 43.4%; from a geometrical series, 17.8%, and their ratio as 1: 2.44. We have, then, a coarsely approximate geometric series, but one which presumably is fairly constant in different individuals.

In the application of the method of the psychophysic series to the time-sense, it was found that the first set of those subjects upon whom more than one set was taken, conformed much more closely to a geometrical series than did the following ones. The same is true of only one of the three subjects who contributed two sets to the present study. For W. D. B. the ratio of approximation to a geometrical series to that of an arithmetical series, as in his first set, 1: 5.27; in his second set, 1: 1.60; in the two combined, 1: 2.86; for G. W. M. these ratios are 1: 2.83; 1: 2.42; and 1: 2.97; for J. J., 1: 1.63; 1: 1.90; and 1: 2.17. This would indicate that practice has less tendency to change the method of judging lifted weights than of time-intervals; in the latter case the approximation to a geometric series is much closer than in the former.

It will be observed that the deviation from a geometric series proceeds, not in a hap-hazard way, but exhibits a fairly definite and constant tendency. The ratio between the average weights of neighboring magnitudes is not a constant, but decreases by smaller and smaller steps, and thus approaches a constant. The unusually high ratio between the classes or magnitudes, I.-II. is a common characteristic of such results (see this Journal, Vol. I. p. 123, Vol. IV. p. 216) and is in large measure accounted for by the fact that the number and average weight of observations falling in Class I. are affected by there being no class smaller than I., to which doubtful judgments might be assigned. The decline of the ratios was in the case of the star-magnitudes expressed by an empirical formula, making the ratio a constant multiplied by a constant times the excess of the magnitude, above a given magnitude. The ratio is expressed, not by a straight line, as it would be if it were a constant, but by a line inclined to the horizontal at a slight angle. A similar inclination, though not a constant one, is suggested by the present results.

The suggestions offered in former applications of this method are entirely corroborated by the present study; the tendency to have equal ratios of objective stimuli correspond to equal sensation-differences is strong and natural in such types of sensation as are estimated grossly and from an impressionist point of view, without reducing them to units or conceiving them as thus reduced. We are quite likely to gauge weights by an unanalyzed feeling of effort, which we do not tend to reduce to pounds and ounces, and this is the natural basis of the psychophysic law. We reserve for a future contribution the general discussion of all the results thus far obtained in the application of the method of the psychophysic series to various types of sensation.

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[1] In this case there are reasons for believing that the subject took a very artificial view of the problem before him, and more or less consciously favored the arithmetical series.

### NOTE UPON OTHER RESEARCHES.

To complete the account of the studies of the year, mention may be made of a few studies, as yet incomplete, or to be published elsewhere. In collaboration with Mr. Geo. W. Moorehouse, a new æsthesiometer has been devised, which differs in essential points from those now in use. It permits of testing the sensibility of the skin with a variable pressure upon the points of the skin tested; the motion by which the points are applied is constant, regular and simple. Furthermore, a series of attachments is to be constructed by which the same apparatus may be used for exploiting the various types of tactile sensibility, for the pressure sense, and for the temperature sense; the apparatus will thus test all the chief sensibilities of the skin. The construction is not elaborate and the cost will be moderate.

Two researches of a statistical nature have been undertaken and are nearing completion. One is a study of the dreams of the deaf, with a view of determining the effect of the age of becoming deaf upon the future retention of "dream hearing," and of recording many other peculiarities of the dreams of this class. Mr. E. T. Johnson has had charge of the tabulation of this interesting but troublesome material. The other is a study of association and community of thoughts; the main point being to determine in what degree different persons are apt to think of the same association when starting from a common point, and then following their own line of association. A word is given to a class of students, and at the same time each member of the class writes the first five words suggested by the original word. The proportion of similarity of association in all the first words written, in all the second, etc., as well as in the sum total of all the words is the chief point to be studied; and the main result is the regular decrease of community of association as the words are removed from the original word. The first words suggested to different persons by a given word are more apt to be the same than the second, the second more so than the third, and so on.